Integrable System Constructed out of Two Interacting Superconformal Fields

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Abstract

We describe how it is possible to introduce the interaction between superconformal fields of the same conformal dimensions. In the classical case such construction can be used to the construction of the Hirota - Satsuma equation. We construct supersymmetric Poisson tensor for such fields, which generates a new class of Hamiltonin systems. We found Lax representation for one of equation in this class by supersymmetrization Lax operator responsible for Hirota - Satsuma equation. Interestingly our supersymmetric equation is not reducible to classical Hirota - Satsuma equation. We show that our generalized system is reduced to the one of the supersymmetric KDV equation (a=4) but in this limit integrals of motion are not reduced to integrals of motion of the supersymmetric KdV equation.

1 Introduction.

The Korteweg - de Vries equation is probably most popular soliton equation which have been extensively studied by mathematicians as well as by physicists [1] in the last 30 years. Beside supplying nice example of the completely integrability of this equation, it bears a deep relation to conformal field theory [2], 2D gravity and matrix models [3].

A remarkable feature of the KdV hierarchies is its relation, via the second Hamiltonina structur to the Virasoro algebra discovered by Gervais [4] . This observation has been extended to other Lie algebras also, as for example the Nonlinear Schrodinger equation is connected with the $SL(2, C)$ Kac -Moody algebra [5] and the Boussinesq equation connected with the so called W_2 algebra [6].

On the other side many different generalizations of the soliton equation have been proposed recently as the Kadomtsev-Petviashvilli, Gelfand - Diki hierarchies and supersymmetrization. The motivations for studing these are diverse, for example in supersymmetric generalization, it is the observation, that in the so called bosonic limit of supersymmetry (*susy*), sometimes we obtain a new class of the integrable models. Up to now, supersymmetric KdV hierarchies [7-16] have been constructed for $N = 1, 2, 3$ and 4 based on relation to superconformal algebras. For extended $N = 2$ supersymmetric case the Boussinesq [17-18], Nonlinear Schrodinger equation [5,19-21] and multicomponent Kodomtsev - Petviashvilli hierarchy [22] have been supersymmetrized as well.

It appeared that in order to get a supersymmetric *susy* theory we have to add to a system of k bosonic equations kN fermions and $k(N-1)$ boson fields $(k = 1, 2, ..., N = 1, 2, ...)$ in such a way that final theory becomes *susy* invariant. Interestingly enough, it appeared that during the supersymmetrizations, some typical *susy* effects (compare to the classical theory) occured. We mention a few of them : the nonuniqueness of the roots for *susy* Lax operator [15], the lack of the bosonic reduction to classical equations (for example in *susy* Boussinesq equation [17]) and the occurence of non-local conservation laws [23]. These effects rely strongly on the descriptions of the generalized systems of equations which we would like to investigate.

In this letter we would like to study problem how it is possible to build the Hamiltonian operator (Poisson tensor) and integrable system using two interacting between themselves (super)conformal fields. That it is possible

to carry out such construction (susy) Boussinesq equation is good example, where we have two conformal fields with different conformal dimensions. However we are interseting in construction where we use two different fields with the same conformal dimensions.

First we study classical aspect of our problem without any reference to supersymmetry and next we consider its supersymmetrizations.

In the "classical" part we show that it is possible to construct several different Poisson tensors using two conformal fiels of same dimensions. We carried out this construction assuming that in limiting case when second field vanishes our Poisson tensor reduces to Poisson tensor which is connected with Virasoro algebra and hence reproduces Korteweg-de Vries equation. Among these different Poisson tensors there is tensor which could be used to construction of Hirota-Satsuma equation [24]. This equation is a nontrivial extension of Korteweg - de Vries equation which is integrable, possesses Lax operator [25] and has the recursion operator [26]. Moreover in limiting case of pure KdV equation (when second field vanishes) integrability is preserved and Lax operator is reduced to KdV counterparts.

In supersymmetric case, presented in second part, we have much more complicated situation compare to classical one. First we carried out classifications of all possible supersymmetric Poison tensor constructed out of two superconformal fields of the same dimensions. We used in this aim the symbolic computer language Reduce [27] and computer package SUSY2 [28]. Similarily to the classical case we assumed that these tensors should be reducible to tensors connected with the $N = 2$ super Virasoro algebra and hence to those which reproduces *susy* generalizations of KdV equation. The $susy(N = 2)$ extension of KdV equation is a class of equations containing one free parameter, however only three members of this class $a = 1, 4, -2$ are completely integrable and possesess Lax pairs. Therefore using once more computer and package SUSY2 we investigate Lax operator. We assumed the most general ansats on Lax operator, constructed out of two superconformal fields, assuming that it reduces to known Lax operators of $susyKdV$ equations. We showed that it reproduces consistent equation only for system which is reduced to $susyKdV(a = 4)$ equation. Interestingly our Lax operator for that system reduces to very simple form. Finally we present three nontrivial Hamiltoninans for our system.

As the result, in the bosonic limit of our system, we obtained a complicated system of four interacting classical fields. Surprisingly these equations

are not reduced to classical Hirota - Satsuma equation. We are not surprised, because as we mentioned earlier for super extension of Boussinesq equation we encounter the same situation - the lack of the proper bosonic limit. There is also second aspect of our supersymmetrization: namely conservations laws of our super system does not coincide, in the limit of pure $susyKdV$ equation, with the conservations laws of $susy\ KdV$ $(a = 4)$ equation. We proved it by showing the absence of the integrals of motions of second, fourth and sixth conformal dimension in our generalization. Let us remark that our generalization which could be considered as the supersymmetrization of the Hirota-Satsuma equation is integrable due to the existence of Lax operator.

2 Classical Poison tensor and Hirota - Satsuma equation

Let us start our consideration noticing that famous Korteweg - de Vries equation

$$u_t = -u_{xxx} + 6uu_x, \tag{1}$$

can be treated as a Hamiltonin system

$$u_t = \{u, H\}, \tag{2}$$

with the Hamiltoninan and the Poisson brackets defined by

$$H = \frac{1}{2} \int u^2 dx, \tag{3}$$

$$\{u(x), u(y)\} = (-\partial^3 + 2u\partial + 2\partial u)\delta(x - y). \tag{4}$$

For later pourpose let us rewrite this equation in the equivalent form using the Poisson tensor

$$P_2 = -\partial^3 + 2u\partial + 2\partial u, \tag{5}$$

$$u_t = P_2\ grad(H), \tag{6}$$

where $grad$ denotes the functional gradient.

For the Fourier modes of $u(x)$,

$$u(x) = \frac{6}{c} \sum_{n=-\infty}^{\infty} exp(-inx)L_n - \frac{1}{4}, \tag{7}$$

the Poisson brackets in eq. (4) imply the structure relations of the Virasoro algebra

$$[L_n, L_m] = (n-m)L_{n+m} + cn(n^2-1)\delta_{n,m}, \tag{8}$$

where c is a central extension term.

It is well known that this equation is completely integrable with infinite numbers of integrals of motion being in invotution among themselves. The interesting problem in theory of solitons is to generalize the KdV equation to system of equations, in such a way, that to preserve integrability and in limiting case, where additional fields vanishes, to recover the usual Korteweg de Vries equation. At the moment we have many different proposal and one of them is the utilization of Poisson tensor constructed out of two different conformal fields u and w of same conformal dimensions. Taking into account that the field u is two dimensional, while usual Poisson tensor of KdV equation is three dimensional, we make the following ansatz

$$P_2 = \begin{pmatrix} c_1\partial_x^3 + z_1 kd(u) & c_2\partial_x^3 + z_2 kd(u) + z_3 kd(w) \\ c_2\partial_x^3 + z_2 kd(u) + z_3 kd(w) & c_3\partial_x^3 + z_4 kd(u) + z_5 kd(w) \end{pmatrix}, \tag{9}$$

where $c_1, ..z_1, ...$ are at the moment free coefficients and

$$kd(u) = u\partial_x + \partial_x u. \tag{10}$$

In order to obtain the conditions on the coefficients c_i and z_i we verified the Jacobi identity [29]

$$< a, P^{'}[Pb]c > + cyclic\ permutation\ of\ \ (a,b,c) = 0, \tag{11}$$

where $P^{'}[Pb]$ denotes the directional derivative along Pb and $<,>$ is a scalar product while a, b, c are arbitrary test functions. We obtained three different solutions on the coefficients c_i and z_i

$$z_2 = z_3 = z_4 = c_2 = 0, \tag{12}$$

$$z_3 = 0, \quad z_5 = \frac{z_2^2 - z_1 z_4}{z_2}, \quad c_1 = \frac{c_2 z_1}{z_2}, \tag{13}$$

$$z_2 = 0, \quad z_1 = z_3, \quad c_3 = \frac{c_1 z_4 + c_2 z_5}{z_3}, \tag{14}$$

The first solution give us the direct product of two standard Virasoro structures eq.(5) with arbitrary central charges c_1 and c_3. We can apply this Poisson tensor to gradient of

$$H = \frac{1}{2}\int uw dx, \tag{15}$$

and obtain equation which have been considered in [] in context of the extended supersymmetric ($N = 3$) KdV system. We will not consider further such possibilities and concentrate our attention on other solutions.

The second solutions is not interesting from our point of view because it impossible to reduced, in the usual manner, this tensor to the standard Virasoro type Poisson tensor. Indeed in order to see it let us brifly explain standard Dirac reduction [6] formula.

Let U, V be two linear spaces with coordinates u and v. Let

$$P(u,v) = \begin{pmatrix} P_{uu}, & P_{uv} \\ P_{vu}, & P_{vv} \end{pmatrix}, \tag{16}$$

ba a Poisson tensor on $U \bigoplus V$. Assume that P_{vv} is invertible, then

$$P = P_{uu} - P_{uv}P_{vv}^{-1}P_{vu}, \tag{17}$$

is a Poisson tensor on U.

As we see the reduction, for second solution, in space where $w = 0$ is possible if $c_2 = 0$ and $z_2 = 0$, but then we obtain undefined central extension term. However it is interesting to notice that we can carry out the reduction in different manner also. Indeed we can deform this structure, in the following self-consistent way:

$$w \to z_2 w, \quad \frac{c_2}{z_2} \to k, \quad c_2 \to 0, \quad z_2 \to 0, \tag{18}$$

and obtain desired result. On the other side it is possible to make reduction in space where $u = 0$ assuming that $c_2 = 0$ and obtaining standard Virasoro type tensor for the field w. In the next we will not consider this case also.

The last solution is most interesting which allows us to make reduction in the space $w = 0$ assuming that $c_2 = 0$. This class of Poisson tensor includes the Hamiltonian operator responsible for the Hirota - Satsuma equation which has the form

$$P_2 = \begin{pmatrix} \partial^3 + \partial u + u\partial & \partial w + w\partial \\ \partial w + w\partial & 2\partial^3 + 2\partial u + 2u\partial \end{pmatrix}, \tag{19}$$

From this form of Poisson tensor we see that the interaction of the fields is concentrated in the the diagonal as well as off diagonal elements of tensor. Therefore we can state, that we constructed extended Virasoro algebra which contains usual conformal algebra interacting with additional conformal field.

The Hamiltonian and equations of motion for Hirota -Satsuma system are

$$H = \frac{1}{2} \int u^2 - w^2, \tag{20}$$

$$\begin{aligned} u_t &= u_{xxx} + 3uu_x - 3ww_x \,, & (21) \\ w_t &= -2w_{xxx} - 3uw_x \,. & (22) \end{aligned}$$

Hirota and Satsuma have found [24] five notrivial integrals of motion and latter it was proved that this equation is integrable by presenting its Lax representations [25]

$$L = (\partial^2 + u + w) * (\partial^2 + u - w), \tag{23}$$

$$L_t = [L, (L^{\frac{3}{4}})_+], \tag{24}$$

where $(+)$ denotes the projection onto the pure differentail part of the operator.

3 The extended supersymmetrization of Poisson tensor constructed out of two fields.

The basic objects in the supersymmetric analysis are the superfield and the supersymmetric derivative. We will deal with the so called extended $N = 2$ supersymmetry for which superfields are superfermions or superbosons depending, in addition to x and t, upon two anticommuting variables, θ_1 and θ_2, $(\theta_2\theta_1 = -\theta_1\theta_2, \theta_1^2 = \theta_2^2 = 0)$. Their Taylor expansion with respect to θ is

$$U(x, \theta_1, \theta_2) = u_o(x) + \theta_1\zeta_1(x) + \theta_2\zeta_2(x) + \theta_2\theta_1 u_1(x), \tag{25}$$

where the fields u_o, u_1, are to be interpreted as the boson (fermion) fields for superboson (superfermion) field, while ζ_1, ζ_2, as fermions (bosons) for superboson (superfermion) respectively. The superderivatives are defined as

$$\mathcal{D}_1 = \partial_{\theta_1} + \theta_1\partial \,, \qquad \mathcal{D}_2 = \partial_{\theta_2} + \theta_2\partial, \tag{26}$$

with the properties

$$\mathcal{D}_2\mathcal{D}_1 + \mathcal{D}_1\mathcal{D}_2 = 0\,, \qquad\qquad \mathcal{D}_1^2 = \mathcal{D}_2^2 = \partial. \tag{27}$$

Below we shall use the following notation: $(\mathcal{D}_i F)$ denotes the outcome of the action of superderivative on the superfield, while $\mathcal{D}_i F$ denotes action itself.

The supersymmetric Poisson tensor connected with Virasora algebra has the form

$$P = cD_1D_2\partial + zs(U), \tag{28}$$

$$s(U) = 2\partial U + 2u\partial - D_1UD_1 - D_2UD_2, \tag{29}$$

where c is central extension term, while z an arbitrary free parameter. We assume that in *susy* case the analog of the formula (9) reads

$$P_2 = \begin{pmatrix} c_1D_1D_2\partial + z_1s(U) & c_2D_1D_2\partial + z_2s(U) + z_3s(W) \\ c_2D_1D_2\partial + z_2s(U) + z_3s(W) & c_3D_1D_2\partial + z_4s(U) + z_5s(W) \end{pmatrix}. \tag{30}$$

We checked the Jacobi identity using the same formula as in classical case and got the same conditions on the central extensions terms c_i and z_i as in classical case. From same reasons, as in classical case, we consider last solution only, assuming additionally that $c_2 = 0$. We can easily obtain some Hamiltonian system, using analog of formula (6) in which we consider the most general three conformal dimensional Hamiltonian. Such Hamiltonian has following density

$$\begin{aligned} H = \; & a_1(D_1D_2U)U + a_2(D_1D_2U)W + a_3(D_1D_2W)W + \\ & a_4W^3 + a_5W^2U + a_6WU^2 + a_7U^3 + a_8W_xU/,, \end{aligned} \tag{31}$$

where a_i are an arbitrary coefficients, superboson U is defined by eq.(25) while superboson W is

$$W = w_o + \theta_1\xi_1 + \theta_2\xi_2 + \theta_2\theta_1w_1, \tag{32}$$

where ξ_i are the fermions valued functions, while w_i are classical functions.

In that manner it is possible to obtain a hudge class of complicated Hamiltonian systems, with many free parameters and this which contains the *susy* generalization of the *KdV* equation.

4 The strategy and results.

We have seen in the last section that it is possible to obtain new class of Hamiltonin systems. We would like to find, in this class, the integrabe one and this which contains the Hirota - Satsuma equation in bosonic limit. Therefore we applied following strategy in order to solve this problem:

1.) We assume the equations of motion on the superbosons U and W in the form which is obtained by applications of Poisson tensor (30) with the conditions (14) and $c_2 = 0$ to the gradient of Hamiltonian (31).

2.) We construct most general *susy* generalization of "classical" Lax operator appearing in Hirota-Satsuma equation (23) and investigate supresymmetric generalization of its Lax pair (24).

3.) We use equations of motion constructed in first approach to the verifaction of validity of Lax pair obtained in second approach. In that manner we obtain the system of algebraic equations on the free parametrs which appeare in Lax operator as well as in equations of motion and in Poisson tensor. This system of equation we would like to solve.

Before presenting the results of our computations let us brifly recall basic facts on *susy* $N = 2$ generalizations of the KdV equation which are needed in this construction. This generalization could be written down as

$$\begin{aligned} U_t = \quad & P\,grad(\tfrac{1}{2}U(D_1D_2U) + \tfrac{a}{3}U^3) = \\ & \partial(-U_{xx} + (2+a)U(D_1D_2U) + (a-2)(D_1U)(D_2U) + aU^3)\,, \end{aligned} \tag{33}$$

where P is defined by (28) while a is an arbitrary parameter. It appeared that this *susy* generalisation is integrable only for three values of parametres a. The integrability have been conclude from the observation that it is possible to find Lax operators [10,16] for these cases.

The Lax operator in the supersymmetric case is an element of the super pseudo-differential algebra G which each element g could be presented as

$$G \ni g = \sum_{n=-\infty}^{\infty} \Phi_n \partial^n = \sum_{n=-\infty}^{\infty} (B_n + F_nD_1 + FF_nD_2 + BB_nD_1D_2)\partial^n, \tag{34}$$

where B_i and BB_i are arbitrary superbosons while F_i and FF_i are arbitrary superfermions. In our case of *susy* KdV generalization, Lax operators are given by:

$$
\begin{aligned}
a=-2: L &= \partial^2 + D_1 U D_2 - D_2 U D_1\,, && (35)\\
a=4 \quad : L &= \partial^2 - (D_1 D_2 u) - u^2 + (D_2 u) D_1 - (D_1 U) D_2 - 2U D_1 D_2,\\
&= -(D_1 D_2 + U)^2\,, && (36)\\
a=1 \quad : L &= \partial - \partial^{-1} D_1 D_2 U\,. && (37)
\end{aligned}
$$

For first two cases we have usual Lax pair [10]

$$
\frac{\partial U}{\partial t} = 4[L, L_{+}^{\frac{3}{2}}]\,, \tag{38}
$$

while for the last case we have nonstandard Lax pair [16]

$$
\frac{\partial u}{\partial t} = [L, L_{\leq 1}^{3}]\,, \tag{39}
$$

where $L_{\leq 1}^{3}$ denotes the projection on the subspace of the

$$
P_{\leq 1}(\Gamma) = \sum_{n=1}^{\infty} \Phi_n \partial^n + (F_0 D_1 + FF_0 D_2 + BB_0 D_1 D_2)\,. \tag{40}
$$

We do not consider, in the next, nonstandard representation, because we do not have such for the Hirota-Satsuma equation.

We have odd and even dimensional integrals of motion for $a = 4$ case. Odd integrals contains usual conservations of law of KdV equation while even integrals does not have such property. Explicitely we have first four integrals of motion for $a = 4$ case

$$
\begin{aligned}
I_1 &= \int U dx d\theta_1 d\theta_2\,, && (41)\\
I_2 &= \int U^2 dx d\theta_1 d\theta_2\,, && (42)\\
I_3 &= \int ((D_1 D_2 U) U + \frac{4}{3} U^3) dx d\theta_1 d\theta_2 && (43)\\
I_4 &= \int (U_x^2 + 3(D_1 D_2 U) U^2 + 2U^4) dx d\theta_1 d\theta_2\,. && (44)
\end{aligned}
$$

These integrals could be computed using following formula

$$
\begin{aligned}
I_{2k+1} &= \int Tr L_1^{2k+1} dx d\theta_1 d\theta_2\,, && (45)\\
I_{2k} &= \int Tr (L_1 L_2)^k dx d\theta_1 d\theta_2\,, && (46)
\end{aligned}
$$

where Tr denotes trace formula defined on the algebra of *susy* pseudo-differential algebra G. We use usual definition of Tr as this which denotes element standing before $D_1D_2\partial^{-1}$ in the algebra G. L_1 and L_2 are two different roots of Lax operator (eq. 33) where L_1 has standard form as $\partial + ...$, while L_2 is $D_1D_2 + U$.

In order to construct Lax operator for our generalization we assumed that it has following reprsentation

$$L = \partial^4 + \Phi_3\partial^3 + \Phi_2\partial^2 + \Phi_3\partial + \Phi_4 \,, \tag{47}$$

where Φ_i are *susy* operators of i-th conformal dimension constructed out of all possible combinations of D_1, D_2, D_1D_2, u, w, (*susy*)derivati ves of u, w and with free parameters. It is a hudge expression which contains 243 terms (or in other words 243 free parametrs). We had made two additional assumptions:

First: in the limit $W = 0$ we should recover Lax operator for *susy* KdV equation in the form of eq.35 or eq.36

Second : our ansatz should be $O(2)$ invariant under the change of the supersymmetric derivatives ($D_1 \mapsto -D_2, D_2 \mapsto D_1$). This invariance follows from physical assumption on the nonprivliging the "fermionic" coordinates in the superspace.

These assumptions simplify our ansats on Lax operator giving for $a = 4$ case 208 terms while for $a = -2$ case 195 terms only. After making these simplifications we were able to realized third point in our strategy and it appeared that only for $a = 4$ case we solved our consistency conditions and obtained one nontrivial solution only. Our system of equation could be written down as

$$\frac{d}{dt}\begin{pmatrix} U \\ \\ W \end{pmatrix} = P_2 * grad((D_1D_2U)U + \frac{4}{3}U^3 + (D_1D_2W)W - 2W^2U), \tag{48}$$

where

$$P_2 = \begin{pmatrix} D_1D_2\partial + s(U) & s(W) \\ & \\ s(W) & D_1D_2\partial + s(U) \end{pmatrix}, \tag{49}$$

and $s(U)$, $s(W)$ are defined by (29).

Explicitely we obtained

$$\begin{aligned} U_t \;\; = \;\; & \partial[-U_{xx} + 3(D_1U)(D_2U) + 6(D_1D_2U)U + 4U^3 + \\ & 3(D_2W)(D_1W) - 6W^2U] \,, \end{aligned} \tag{50}$$

$$
\begin{aligned}
W_t &= \partial[-W_{xx} - 2W^3 + 3(D_2W)(D_1U) - 3(D_1W)(D_2U)] - \\
&\quad 6(D_2W)(D_2U)U - 6(D_1W)(D_1U)U\,.
\end{aligned} \tag{51}
$$

In the bosonic sector we obtained

$$
u_{ot} = \partial[-u_{oxx} + 6u_1u_o + 4u_o^3 - 6w_ou_o], \tag{52}
$$

$$
w_{ot} = \partial[-w_{oxx} - 2w_o^3], \tag{53}
$$

$$
\begin{aligned}
u_{1t} &= \partial[-u_{1xx} + 3u_1^2 + 3w_1^2 + 3w_{ox}^2 - 3u_{ox}^2 - 6u_{oxx}u_o + \\
&\quad 12u_1u_o^2 - 6w_o^2u_1 - 12w_1w_ou_o]\,,
\end{aligned} \tag{54}
$$

$$
\begin{aligned}
w_{1t} &= \partial[-w_{1xx} + 6w_{ox}u_{ox} + 6w_1u_1 - 6w_1w_o^2] \\
&\quad +12w_1u_ou_{ox} - 12w_{ox}u_1u_o\,.
\end{aligned} \tag{55}
$$

Interestingly our Lax operator has simple representation

$$
L := [(D_1D_2 + U + W)(D_1D_2 + U - W)]^2\,, \tag{56}
$$

This form of Lax operator suggests to consider much simpler Lax pair, namelly it is enought to investigate the root of this Lax operator

$$
L = (D_1D_2 + U + W)(D_1D_2 + U - W), \tag{57}
$$

with corresponding Lax pair

$$
\frac{d\,L}{d\,t} = -4i[L, (L^{\frac{3}{2}})_+]. \tag{58}
$$

If we further reduce bosonic limit of our system of equation, demanding that $u_o = 0$ and $w_o = 0$ we obtain the following system

$$
u_{1t} = \partial(-u_{1xx} + 3u_1^2 + 3w_1^2)\,, \tag{59}
$$

$$
w_{1t} = \partial(-w_{1xx} + 6u_1w_1)\,, \tag{60}
$$

which does not coincides with the Hirota-Satsuma equation eq. (21 - 22). Moreover we can transform last equations to the system of noninteracting two Korteweg - de Vries equations using

$$
u_1 \mapsto u_1 + w_1\,, \tag{61}
$$

$$
w_1 \mapsto u_1 - w_1\,. \tag{62}
$$

However we can not state the same on the supersymmetric level.

It is rather unexpected result because our supersymmetrization method used supersymmetrizations of the Hirota - Satsuma Lax operator. The observation that in the process of the supersymmeytrization we do not obtain, in bosonic limit, the desired equation, is known in the theory of supersymmetrization of soliton's equation. It happen for example in *susy* Boussinesq equation.

Finally let us discuss the problem of existence of the integrals of motion in our model. We successed to construct first three conservations of laws which are

$$
\begin{aligned}
I_1 &= \int U dx d\theta_1 d\theta_2 \,, && (63)\\
I_3 &= \int ((D_1D_2U)U + \frac{4}{3}U^3 + (D_1D_2W)W - 2W^2U) dx d\theta_1 d\theta_2 \,, && (64)\\
I_5 &= \int (16U^5 + 40(D_1D_2U)U^3 + 10(D_1D_2U)^2U + 30U_x^2U - && (65)\\
& 5(D_1D_2U_{xx})U - 10(D_1D_2W)W^3 - 5(D_1D_2W_{xx})W + \\
& 20(D_2W_x)(D_2W)U + 20(D_1W_x)(D_1W)U + 50(D_2W)(D_1W)U^2 + \\
& 20W_{xx}WU + 30(D_1D_2W)^2U - 30(D_1D_2W)WU^2 + 20W_x^2U + \\
& 30W^4U - 3(D_1D_2U)W^2U - 40W^2U^3) dx d\theta_1 d\theta_2,
\end{aligned}
$$

Moreover we proved the absence of integrals of motion of second,fourth and sixth conformal dimensional in our system. It is also unexpected result. It means that our equations of motion does not coincide, in the limit when $W = 0$ with *susy* version of KdV equation, because as we saw the last one possesses odd and even conformal dimensional integrals of motions. In order to explain this situation let us make two observation:

First: We introduced interaction by including second fields and this field destroys "half" of integrals of motion for *susy kdV* equation.

Second: For pure $susyKdVa = 4$ case Lax operator possesses two nonequivalent roots. These roots are responsible for the integrals of motion. In our case we have no such situation and we have one root of Lax operator only.

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